eLearning

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Topic	: **MATRICES-INTRODUCTION & BASIC OPERATIONS**
Subject Code	: HBMA18A01
Subject Name	: ALLIED MATHEMATICS-I
Delivered by	: Dr.P.Sona

MATRIX

A matrix is a rectangular arrangement of numbers or functions which are represented as….

$$A = \begin{bmatrix} a_{11} & a_{12} & .. & .. & .. & a_{1n} \\ a_{21} & a_{22} & .. & .. & .. & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ .. & .. & .. & .. & .. & .. \\ a_{m1} & a_{m2} & .. & .. & .. & a_{mn} \end{bmatrix}$$

- Matrix is enclosed by [ ] *or* { }
- Matrices is plural for matrix
- It is also represented as $A = [a_{ij}]$,
  where $i = 1,2 \ldots . m$ & $j = 1,2 \ldots . n$

ORDER OF A MATRIX

- The order or size or dimension of a matrix is defined by the number of rows and columns it contains

- For a matrix with m rows and n columns, order is $m \times n$, read as 'm by n'

$$If\ A = \begin{bmatrix} 12 & 10 \\ 3 & 12 \\ 5 & 8 \end{bmatrix}, then\ order\ of\ the\ matrix\ A\ is\ \ldots.$$

Number of Rows= 3

Number of Columns= 2

So order of the Matrix A is 3×2

Types of Matrices

Row Matrix

A **row matrix** is a matrix with only one row.

- A is a row matrix of order 1 × 4

$$A = \begin{bmatrix} \mathbf{1} & \mathbf{-3} & \mathbf{1} & \mathbf{4} \end{bmatrix}$$

- B is a row matrix of order 1 × 3

$$B = \begin{bmatrix} \mathbf{4} & \mathbf{5} & \mathbf{1} \end{bmatrix}$$

Column Matrix

A **column matrix** is a matrix with only one column.

- A is a column matrix of order 2 × 1

$$A = \begin{bmatrix} \mathbf{-1} \\ \mathbf{4} \end{bmatrix}$$

- C is a column matrix of order 3 × 1

$$C = \begin{bmatrix} \mathbf{0} \\ \mathbf{4} \\ \mathbf{2} \end{bmatrix}$$

Zero matrix

A **zero matrix** or a **null matrix** is a matrix that has all its elements zero.

- N is a zero matrix of order 2 × 3

$$N = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Square Matrix

A **square matrix** is a matrix with an equal number of rows and columns.

❖ K is a square matrix of order 2 × 2

$$K = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{2} \end{bmatrix}$$

❖ V is a square matrix of order 3 × 3

$$V = \begin{bmatrix} 0 & 4 & 0 \\ 5 & 6 & 1 \\ 3 & 0 & -3 \end{bmatrix}$$

Diagonal Matrix

A **diagonal matrix** is a square matrix that has all its elements zero except for those in the diagonal from top left to bottom right; which is known as the **leading diagonal** of the matrix.

❖ D is a diagonal matrix

$$D = \begin{bmatrix} \mathbf{2} & 0 & 0 \\ 0 & \mathbf{3} & 0 \\ 0 & 0 & \mathbf{7} \end{bmatrix}$$

Scalar Matrix

A **scalar matrix** is a diagonal matrix where all the diagonal elements are equal.

❖ G is a diagonal matrix

$$G = \begin{bmatrix} \mathbf{2} & 0 & 0 \\ 0 & \mathbf{2} & 0 \\ 0 & 0 & \mathbf{2} \end{bmatrix}$$

Unit Matrix

A **unit matrix** or **identity matrix** is a diagonal matrix whose elements in the diagonal are all ones.

- I is a unit matrix

$$I = \begin{bmatrix} \mathbf{1} & 0 & 0 \\ 0 & \mathbf{1} & 0 \\ 0 & 0 & \mathbf{1} \end{bmatrix}$$

Triangular Matrix

Lower Triangular   Upper Triangular

- A triangular matrix is a special kind of square matrix
- A square matrix is called a **lower triangular** if all the entries above the main diagonal are zero

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{4} & \mathbf{-3} & \mathbf{0} \\ \mathbf{5} & \mathbf{6} & \mathbf{2} \end{bmatrix}$$

- A square matrix is called an **upper triangular** if all the entries below the main diagonal are zero

$$\begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{4} \end{bmatrix}$$

Transpose Matrix

Given matrix A, the **transpose of matrix** A is another matrix where the elements in the columns and rows have switched. In other words, the rows become the columns and the columns become the rows.

$$A = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{-4} \\ \mathbf{5} & \mathbf{6} & \mathbf{2} \\ \mathbf{6} & \mathbf{7} & \mathbf{2} \end{bmatrix} \quad A^T = \begin{bmatrix} \mathbf{1} & \mathbf{5} & \mathbf{6} \\ \mathbf{3} & \mathbf{6} & \mathbf{7} \\ \mathbf{-4} & \mathbf{2} & \mathbf{2} \end{bmatrix}$$

Orthogonal Matrix

A square matrix A is called an **orthogonal matrix** if all the product of the matrix A and the transpose matrix A^T is an identity matrix.

$$AA^T = I$$

Symmetric Matrix

A square matrix A is called **symmetric** if $A = A^T$.

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{4} \\ \mathbf{3} & \mathbf{5} & \mathbf{1} \\ \mathbf{4} & \mathbf{1} & \mathbf{3} \end{bmatrix} \qquad A^T = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{4} \\ \mathbf{3} & \mathbf{5} & \mathbf{1} \\ \mathbf{4} & \mathbf{1} & \mathbf{3} \end{bmatrix}$$

- ❖ A is symmetric ⟺ $a_{ij} = a_{ji}$, for all i&j

Skew-Symmetric Matrix

A square matrix A is called **skew-symmetric** if $A = -A^T$.

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{-3} & \mathbf{4} \\ \mathbf{3} & \mathbf{0} & \mathbf{1} \\ \mathbf{-4} & \mathbf{-1} & \mathbf{0} \end{bmatrix} \qquad A^T = \begin{bmatrix} \mathbf{0} & \mathbf{3} & \mathbf{-4} \\ \mathbf{-3} & \mathbf{0} & \mathbf{-1} \\ \mathbf{4} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

- ❖ A is symmetric ⟺ $a_{ij} = -a_{ji}$, for all i&j

Check your understanding!

1. $\begin{bmatrix} \mathbf{3} & 0 & 0 \\ 0 & \mathbf{4} & 0 \\ 0 & 0 & \mathbf{3} \end{bmatrix}$ is a ________ matrix.

2. $\begin{bmatrix} \mathbf{0} & \mathbf{-7} & \mathbf{4} \\ \mathbf{7} & \mathbf{0} & \mathbf{2} \\ \mathbf{-4} & \mathbf{-2} & \mathbf{0} \end{bmatrix}$ is a ________ matrix.

3. Is [2] a column matrix?

Some Basic Operations of Matrices

- ❖ Addition of Matrices
- ❖ Subtraction of Matrices
- ❖ Scalar multiplication of a Matrix
- ❖ Multiplication of Matrices

Addition of Matrices

If A and B be two matrices of the same order, then their sum, A+B is defined as the matrix, each element of which is the sum of the corresponding elements of A and B.

$$A = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 0 \end{bmatrix}$$

Properties of Matrix Addition

Let A, B & C are matrices of same order.

(i) Commutative Law

$$A + B = B + A$$

(ii) Associative Law

$$A + (B + C) = (A + B) + C$$

Subtraction of Matrices

The difference of two matrices is a matrix, each element of which is obtained by subtracting the elements of the second matrix from the corresponding elements of the first.

$$A - B = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 3 & 0 \end{bmatrix}$$

Scalar multiplication of a Matrix

If a matrix is multiplied by a scalar quantity k, then each element is multiplied by k.

$$A = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 & 0 \\ 6 & -2 \end{bmatrix}$$

Multiplication of Matrices

The product of two matrices A and B is only possible if the number of columns A is equal to the number of rows in B.

Let A= $[a_{ij}]$ be an $\boldsymbol{m} \times \boldsymbol{n}$ matrix and B= $[b_{ij}]$ be an $\boldsymbol{n} \times \boldsymbol{p}$ matrix. Then the product of these matrices is an $\boldsymbol{m} \times \boldsymbol{p}$ matrix.

If A= $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix}$, B= $\begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 1 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \times \begin{matrix} C_1 \quad C_2 \\ \begin{bmatrix} -1 & 0 \\ 0 & -2 \\ 1 & 2 \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} R_1C_1 & R_1C_2 \\ R_2C_1 & R_2C_2 \\ R_3C_1 & R_3C_2 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \quad 0 \quad 1) \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} & (1 \quad 0 \quad 1) \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \\ (0 \quad 1 \quad 2) \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} & (0 \quad 1 \quad 2) \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \\ (0 \quad 3 \quad 2) \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} & (0 \quad 3 \quad 2) \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times (-1) + 0 \times 0 + 1 \times 1 & 1 \times 0 + 0 \times (-2) + 1 \times 2 \\ 0 \times (-1) + 1 \times 0 + 2 \times 1 & 0 \times 0 + 1 \times (-2) + 2 \times 2 \\ 0 \times (-1) + 3 \times 0 + 2 \times 1 & 0 \times 0 + 3 \times (-2) + 2 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ 2 & 2 \\ 2 & -2 \end{bmatrix}$$

Properties of Matrix Multiplication

Let A, B & C are matrices of same order.

(i) Multiplication of matrices is not commutative

$$AB \neq BA$$

(ii) Multiplication of matrices is associative

$$A(BC) = (AB)C$$

(iii) Matrix multiplication is distributive with respect to addition

$$A(B + C) = AB + AC$$

(iv)Multiplication of matrix A by unit matrix

$$AI = IA = A$$

Check your understanding!

1. If $A + B = \begin{bmatrix} \boldsymbol{x} & \mathbf{0} \\ \mathbf{3} & \mathbf{-1} \end{bmatrix} + \begin{bmatrix} \mathbf{4} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{5} & \mathbf{2} \\ \mathbf{3} & \mathbf{0} \end{bmatrix}$, then $x =$?

2. If $A = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{3} & \mathbf{-1} \end{bmatrix}$, $B = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{0} & \mathbf{5} \end{bmatrix}$, then find AB and order of AB.

3. If $A = \begin{bmatrix} 1 & -7 & 9 \\ 5 & 4 & 3 \\ -2 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 1 & 0 \\ -3 & 4 & 6 \\ -5 & -9 & 4 \end{bmatrix}$, then find A+B, A-B & 2A, -5B& AB.