eLearning
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Topic	:	**INVERSE OF A MATRIX**
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INVERSE OF A MATRIX

Let A be a matrix of order n. Then matrix B, if it exists, such that AB=BA $=I_n$ is called inverse of matrix A .

$A^{-1} = \frac{1}{|A|}$Adj A

Note:A^{-1} exists if A is non singular. (i.e) $|A| \neq 0$

Adj A= [Cofactor matrix]^T

- **Find the inverse of A $= \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix}$.**

  Solution:

  Given $A = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix}$

  $A^{-1} = \frac{Adj\ A}{|A|}$

  Consider $|A| = \begin{vmatrix} 2 & -1 \\ -4 & 2 \end{vmatrix} = 4 - 4 = 0$

  $\therefore A^{-1}$ does not exist.

- **Find the inverse of A $= \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}$.**

  Solution:

  Given A $=\begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}$

  $A^{-1} = \frac{Adj\ A}{|A|}$

  Consider $|A| = \begin{vmatrix} -1 & 2 \\ 1 & -4 \end{vmatrix} = 4 - 2 = 2 \neq 0$

  $\therefore A^{-1}$ Exist.

  $Adj\ A = \begin{bmatrix} -4 & -2 \\ -1 & -1 \end{bmatrix}$

  $\therefore$ Inverse of A $= A^{-1}$

  $$= \frac{1}{2}\begin{bmatrix} -4 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -1/2 & -1/2 \end{bmatrix}$$

- **If A $= \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}$ verify the result A (adjA) = (adjA) A = $|A|$ I_2 .**

  Solution:

  adjA= $\begin{bmatrix} -4 & -2 \\ -1 & -1 \end{bmatrix}$,$|A| = \begin{vmatrix} -1 & 2 \\ 1 & -4 \end{vmatrix} = 4 - 2 = 2$

$(\text{adj}A)A = \begin{bmatrix} -4 & -2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ ---------------(1)

$A(\text{adj}A) = \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} -4 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ ---------------(2)

$|A|I_2 \quad = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ ---------------(3)

From (1),(2)&(3) we get

$A\,(\text{adj}A) = (\text{adj}A)\,A = |A|\,I_2$

- **If $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify $A\,(\text{adj}A) = (\text{adj}A)\,A = |A|I$.**

Solution:

$\text{adj}A = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}, |A| = \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$

$(\text{adj}A)A = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$ ---------------(1)

$A(\text{adj}A) = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$ ---------------(2)

$|A|I \quad = -11\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$ ---------------(3)

From (1),(2)&(3) we get

$A\,(\text{adj}A) = (\text{adj}A)\,A = |A|I$

- **Find the adjoint of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 2 & 3 \\ 0 & 5 & 3 \end{bmatrix}$.**

Solution:

Adj A= [Cofactor matrix]^T

The co factors are given by

Co factor of 2= $A_{11} = +\begin{vmatrix} 2 & 3 \\ 5 & 3 \end{vmatrix} = 6 - 15 = -9$

Co factor of 2= $A_{12} = -\begin{vmatrix} 1 & 3 \\ 0 & 3 \end{vmatrix} = -3 - 0 = -3$

Co factor of 1= $A_{13} = +\begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix} = 5 - 0 = 5$

Co factor of 1= $A_{21} = -\begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = -(6 - 5) = -1$

Co factor of 2= $A_{22} = +\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 6 - 0 = 6$

Co factor of 3= $A_{23} = -\begin{vmatrix} 2 & 2 \\ 0 & 5 \end{vmatrix} = -10 - 0 = -10$

Co factor of 0= $A_{31} = +\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 6 - 2 = 4$

Co factor of 5= $A_{32} = -\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -(6 - 1) = -5$

Co factor of 3= $A_{33} = +\begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} = 4 - 2 = 2$

$$\therefore \text{ Cofactor matrix } = \begin{bmatrix} -9 & -3 & 5 \\ -1 & 6 & -10 \\ 4 & -5 & 2 \end{bmatrix}$$

$$\therefore \text{Adj A} = [\text{Cofactor matrix}]^T = \begin{bmatrix} -9 & -1 & 4 \\ -3 & 6 & -5 \\ 5 & -10 & 2 \end{bmatrix}$$

- **Find the adjoint of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$.**

Solution:

Adj A= [Cofactor matrix]^T

The co factors are given by

Co factor of 1= $A_{11} = +\begin{vmatrix} 2 & -3 \\ -1 & 3 \end{vmatrix} = 6 - 3 = 3$

Co factor of 1= $A_{12} = -\begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} = -(3 + 6) = -9$

Co factor of 1= $A_{13} = +\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5$

Co factor of 1= $A_{21} = -\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = -(3 + 1) = -4$

Co factor of 2= $A_{22} = +\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3 - 2 = 1$

Co factor of -3= $A_{23} = -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -(-1 - 2) = 3$

Co factor of 2= $A_{31} = +\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3 - 2 = -5$

Co factor of -1= $A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} = -(-3 - 1) = 4$

Co factor of 3= $A_{33} = +\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1$

$$\therefore \text{ Cofactor matrix } = \begin{bmatrix} 3 & -9 & -5 \\ -4 & 1 & 3 \\ -5 & 4 & 1 \end{bmatrix}$$

$$\therefore \text{Adj A} = [\text{Cofactor matrix}]^T = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

- **Find the inverse of A = $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$.**

Solution:

Given $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{vmatrix} = 3(2-0) - 1(-2-0) - 1(4+2)$$

$$= 6 + 2 - 6 = 2 \neq 0$$

$\therefore$ A is non singular & A^{-1} exists.

The co factors are given by

Co factor of 3= $A_{11} = + \begin{vmatrix} -2 & 0 \\ 2 & -1 \end{vmatrix} = 2 - 0 = 2$

Co factor of 1= $A_{12} = - \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = -(-2) = 2$

Co factor of -1= $A_{13} = + \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 4 + 2 = 6$

Co factor of 2= $A_{21} = - \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -(-1+2) = -1$

Co factor of -2= $A_{22} = + \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} = -3 + 1 = -2$

Co factor of 0= $A_{23} = - \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = -(6-1) = -5$

Co factor of 1= $A_{31} = + \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 0 - 2 = -2$

Co factor of 2= $A_{32} = - \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -(0+2) = -2$

Co factor of -1= $A_{33} = + \begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} = -6 - 2 = -8$

$$\therefore Cofactor\ matrix = \begin{bmatrix} 2 & 2 & 6 \\ -1 & -2 & -5 \\ -2 & -2 & -8 \end{bmatrix}$$

$$\therefore \text{Adj A} = [Cofactor\ matrix]^T = \begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{Adj A} = \frac{1}{2}\begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

- **Find the inverse of A = $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.**

Solution:

Given $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = 1(1-1) - 0 + 3(-2-1)$$

$$= -9 \neq 0$$

$\therefore$ A is non singular & A^{-1} exists.

The co factors are given by

Co factor of 1= $A_{11} = +\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 0$

Co factor of 0= $A_{12} = -\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = -(2+1) = -3$

Co factor of 3= $A_{13} = +\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2-1 = -3$

Co factor of 2= $A_{21} = -\begin{vmatrix} 0 & 3 \\ -1 & 1 \end{vmatrix} = -(0+3) = -3$

Co factor of 1= $A_{22} = +\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 1-3 = -2$

Co factor of -1= $A_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -(-1) = 1$

Co factor of 1= $A_{31} = +\begin{vmatrix} 0 & 3 \\ 1 & -1 \end{vmatrix} = 0-3 = -3$

Co factor of-1= $A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -(-1-6) = 7$

Co factor of 1= $A_{33} = +\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1-0 = 1$

$\therefore$ Cofactor matrix $= \begin{bmatrix} 0 & -3 & -3 \\ -3 & -2 & 1 \\ -3 & 7 & 1 \end{bmatrix}$

$\therefore$ Adj A= $[\text{Cofactor matrix}]^T = \begin{bmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}\text{Adj A} = \frac{1}{-9}\begin{bmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{bmatrix}$

Check your understanding!

1. Find the inverse of A = $\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{3} \\ \mathbf{2} & \mathbf{1} & \mathbf{-1} \end{bmatrix}$

2. Find the inverse of A = $\begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{3} \\ \mathbf{-5} & \mathbf{3} & \mathbf{1} \\ \mathbf{-3} & \mathbf{2} & \mathbf{3} \end{bmatrix}$

3. Find the adjoint of the matrix $\begin{bmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{1} \\ \mathbf{-15} & \mathbf{6} & \mathbf{-5} \\ \mathbf{5} & \mathbf{-2} & \mathbf{2} \end{bmatrix}$