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Topic	: **INFERENCE THEORY-UNIT-1**
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The Theory of Inference

Inference Theory is concerned with the inferring of a conclusion from certain hypothesis or basic assumptions, called as premises, by applying certain principles of reasoning, called as rules of inference. When a conclusion derived from a set of premises by using the rule of inference, the process of such derivation is called a formal proof. The rules of inference are only means used to draw a conclusion from a set of premises in a finite sequence of steps, called argument.

These rules will be given in terms of statements formulae rather than in terms of any specific statements or their Truth values. Any conclusion which is arrived at by following these rules is called a valid conclusion and the argument is called valid argument.

TRUTH TABLE TECHNIQUE

When A and B are two statements formulas, then B is said to (logically) follow A or B is a valid conclusion of the premise A if $A \rightarrow B$ is a tautology. (ie) $A \Rightarrow B$. Extending this idea,

We say that the statement follows logically from the set of Premises $\{H_1, H_2, H_3, \ldots H_n\}$ iff $H_1 \wedge H_2 \wedge H_3 \ldots \wedge H_n \Rightarrow C$

If a set of premises and a conclusion are given, it is possible to determine whether the conclusion follows from the premises by constructing the relevant truth tables. Such method is called truth table technique.

Procedure:

We want to show whether C logically follows from $H_1, H_2, H_3, \ldots H_n$ (i.e) To show $H_1 \wedge H_2 \wedge H_3 \ldots \wedge H_n \rightarrow C$ is a Tautology.

We look for those rows of $H_1, H_2, H_3, \ldots H_n$ which have a truth value T. If for every such row C also has a truth value True then the conclusion C logically follows from $H_1, H_2, H_3, \ldots H_n$

- ***Rule P***

A given Premises may be introduced at any stage in the derivation.

- ***Rule T***

A formula S may be introduced in a derivation if S is tautologically implied by one or more of the preceding formulae in the derivation.

- ***Rule CP(Conditional Proof)***

If we can derive S from R and a set of the given premises then we can derive $R \rightarrow S$ from the set of premises alone. In such a case R is taken as the additional premises. Rule CP is also called as deduction theorem. The analysis of the validity of the formula from the given set of premises by using the derivation is called as “The theory of inferences”.

Example1 : H_1:P, H_2:$P \rightarrow Q$ $C: Q$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

In the first row of the Truth table H_1: P is True, H_2: $P \rightarrow Q$ is True

Notice that C: Q is also True∴P, $P \rightarrow Q \Rightarrow Q$

Example2 : H_1: $P \rightarrow Q$, H_2: $Q \rightarrow R$ C: $P \rightarrow R$

Solution:

To show $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow P \rightarrow R$

Let $(P \rightarrow Q) \wedge (Q \rightarrow R)$ be True

∴ P→ Q is True and $Q \to R$ is True

P→ Q is True imply $\begin{cases} P\ is\ True, Q\ is\ True \ldots\ldots\ldots (i) \\ P\ is\ False, Q\ is\ True \ldots\ldots. (ii) \\ P\ is\ False\ , Q\ is\ False \ldots\ldots (iii) \end{cases}$

$Q \to R$ is True imply $\begin{cases} Q\ is\ True, R\ is\ True \ldots\ldots\ldots (iv) \\ Q\ is\ False, R\ is\ True \ldots\ldots. (v) \\ Q\ is\ False\ , R\ is\ False \ldots\ldots (vi) \end{cases}$

From (i), (iv) P→ R is True. From (ii), (iv) P→ R is True

Hence antecedent True imply consequent to be True

∴ $H_1 \wedge H_2 \Rightarrow C$

Other Rules of Inferences

Rules of Inference	Rules in tautological form	Name of the Rule
P∧Q **∴ P**	**(P∧Q)→P, (P∧Q)→Q**	**Simplification**
P **∴ P∨Q**	**P→(P∨Q), Q→(P∨Q)**	**Addition**
P **Q** **∴ P∧Q**	**((P) ∧ (Q)) → (P∧Q)**	**Conjunction**
P **P→ Q** **∴ Q**	**[P∧(P→ Q)] → Q**	**Modus ponens**
7 Q **P→ Q** **∴ 7P**	**[7Q ∧ (P→ Q)] → 7P**	**Modus tollens**
P→ Q **Q→ R** **∴P→ R**	**[(P→ Q) ∧ (Q→ R)] →(P→ R)**	**Hypothetical Syllogism**
P∨Q **7P** **∴ Q**	**[(P∨Q) ∧ 7P] → Q**	**Disjunctive Syllogism**
P∨Q **7 P∨R** **∴ Q∨R**	**[(P∨Q) ∧ (7P∨R)] → Q∨R**	**Resolution**
	[(P∨Q) ∧ (P→ R)∧(Q → R)] → R	**Dilemma**

Often using Equivalences

1.7(P→ Q) <=> P∧7Q

2. P→ Q <=> 7Q → 7P

3. P↔ Q <=> (P→ Q)∧(Q → P)

4. P↔ Q <=> 7P↔ 7Q

Question1.

Show that R∧(P∨ Q) is a valid conclusion from the premises P∨ Q, Q →R, P→M and 7M

Solution:

Step	Derivation	Rule
1	P→M	P
2	7M→7P	T
3	7M	P

4	7P	T[from 2 &3]
5	P∨ Q	P
6	7P→ Q	T[from 5] P∨ Q <=> 7P→ Q P→ Q <=> 7P∨ Q
7	Q	T [from 4,6]
8	Q →R	P
9	R	T [from 7 & 8]
10	R∧(P∨ Q)	T [from 5 & 9]

Question 2

Show that the following premises are inconsistent

(1) If Jack misses many classes due to illness, then he fails in the high school.
(2) If Jack fails in the high school, then he is uneducated.
(3) If Jack reads a lot of books, then he is not uneducated.
(4) Jack misses many classes due to illness and reads a lot of books

Solution: Let us consider the atomic statements

P: Jack misses many classes due to illness

Q: Jack fails in the high school

R: Jack reads a lot of books.

S: Jack is uneducated.

Then the given premises are: P→Q, Q→S, R→ 7S, P∧R.

Step	Derivation	Rule
1	P→Q	P
2	Q→S	P
3	P→S	T[from 1 &2]
4	7S→ 7P	T[A→ B <=>7B→ 7A, from 3]
5	R→ 7S	P
6	R →7P	T[from 4 & 5]
7	7R∨7P	T [from 6] P→ Q <=>7P∨ Q
8	7(P∧R)	T[from 7 De Morgan law]
9	P∧R	P
10	(P∧R)∧ 7(P∧R)	T [from 8 &9] P∧7P<=>F contradiction

Question 3

Show that J∧S follows logically from the premises P →Q, Q→7R, R and P∨(J∧S)

Solution:

Step	Derivation	Rule
1	P→Q	P
2	Q→ 7R	P
3	P→7R	T[from 1 &2]
4	R→ 7P	T[from 3]
5	R	P

6	7P	T[from 4 & 5]
7	P∨(J∧S)	P
8	7P→(J∧S)	T[from 7]
9	J∧S	T[from 6 & 8]

∴(J∧S) follows logically from the given premises.

Question 4

Show that P →S follows logically from the given premises 7P∨Q, 7Q∨R, and R →S

Solution:

Step	Derivation	Rule
1	7P∨Q	P
2	P→ Q	T[from 1]
3	7Q∨R	P
4	Q→ R	T[from 3]
5	P→ R	T[from 2 & 4]
6	R→ S	P
7	P→ S	T[from 5 & 6]

Question 5

Demonstrate that R is a valid inference from the premises P→ Q, $Q \rightarrow R$ and P

Solution:

Step	Derivation	Rule
1	P→ Q	P
2	P	P
3	Q	T[from 1 & 2, modus ponens]
4	Q→ R	P
5	R	T[from 3 & 4, modus ponens]

Question 6

Show that (a∨b) follows logically from the premises P∨Q, (P∨Q)→ $7R$, 7R→ $(S \wedge 7T)$ *and* $(S \wedge 7T)$ → (a∨b)

Solution:

Step	Derivation	Rule
1	(P∨Q)→ $7R$	P
2	7R→ $(S \wedge 7T)$	P
3	(P∨Q)→ $(S \wedge 7T)$	T[from 1& 2 and hypothetical syllogism]
4	P∨Q	P
5	S∧7T	T[from 3 & 4, modus ponens]
6	$(S \wedge 7T)$ → (a∨b)	P

7	a∨b	T[from 5 & 6, modus ponens]

Question 7

Derive P→ $(Q \to S)$ using the CP rule from the premises P→ $(Q \to R)$, Q→ $(R \to S)$

Solution:

We shall assume P as an additional premise. Using P and the two given premises, we will derive $(Q \to S)$. Then by CP rule P→ $(Q \to S)$ is deemed to have been derived from the two given premises.

Step	Derivation	Rule
1	P	P (additional)
2	P→ $(Q \to R)$	P
3	$Q \to R$	T[from 1 &2,modus ponens]
4	7Q ∨R	T [from 3 and equivalence of 3]
5	Q→ $(R \to S)$	P
6	7Q∨$(R \to S)$	T[from 5 and equivalence of 5]
7	7Q∨$(R \wedge (R \to S))$	T [from 4 ,6 and distributive law]
8	7Q ∨S	T[7, modus ponens]
9	$Q \to S$	T[from 8 and equivalence of 8]
10	P→ $(Q \to S)$	T [from 9and CP rule]