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TOPIC	:	DIFFERENTIATION
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DELIVERED BY	:	MRS G.ANNALAKSHMI
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DIFFERENTIATION

BASIC CONCEPTS OF DIFFERTIATION

Consider a function y=f(x) of a variable x. Suppose x changes from an initial value x_0 to a final value x_1. Then the increment in x defined to be the amount of change in x. It is denoted by Δx. The increment in y namely Δy depends on the values of x_0 and Δx.

If the increment Δy is divided by Δx the quotient $\frac{\Delta y}{\Delta x}$ is called the average rate of change of y with respect to x, as x changes from x_0 to x_0+Δx. The quotient is given by $\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$

This fraction is also called a difference quotient.

Differentiation using standard formulae

- **Define differentiation.**

Solution:

The rate of change of one variable quantity with respect to another variable quantity is called Differentiation.
(i.e)If y is a function of x, then the rate of change of y with respect to x is called the differential co-efficient of y. It is denoted by dy/dx (or) $d(f(x))/dx \quad (or) \ f'(x) \ (or) \ Df(x)$

- **Find $\frac{dy}{dx}$ if $y=3sinx + 4cosx - e^x$.**

Solution:

$$\frac{dy}{dx} = 3cosx \ - 4sinx - e^x$$

****

- **Find $\frac{dy}{dx}$ if y=$e^x + 3tanx + logx^6$.**

Solution: Given $y=e^x + 3tanx + 6logx$

$$\frac{dy}{dx} = e^x + 3\sec^2 x + \frac{6}{x}$$

****

- **If $y=x^3 - 6x^2 + 7x + 6 \ + \cos x + \frac{1}{x\sqrt{x}}$, find $\frac{dy}{dx}$.**

Solution:

Given $y = x^3 - 6x^2 + 7x + 6 + \cos x + x^{\frac{-3}{2}}$

$$\frac{dy}{dx} = 3x^2 - 12x + 7 - sinx + \left(\frac{-3}{2}\right)x^{\frac{-3}{2}-1} = 2x - sinx + \frac{-3}{2x^{\frac{5}{2}}}$$

****

- **If $y = \frac{3}{\sqrt{x}} + 3x^4 - \frac{1}{\sqrt[3]{x}}$, find $\frac{dy}{dx}$.**

<u>Solution:</u> Given $y = \frac{3}{\sqrt{x}} + 3x^4 - \frac{1}{\sqrt[3]{x}}$

$$\frac{dy}{dx} = 3\left(\frac{-1}{2}\right)x^{\frac{-1}{2}} + 12x^3 - \left(\frac{-1}{3}\right)x^{\frac{-4}{3}}$$

$$= \frac{-3}{2x^{\frac{3}{2}}} + 12x^3 + \frac{1}{3x^{\frac{4}{3}}}$$

****

- **Find the derivative of $y = e^{7x} + sin3x + e^{5x+3}$.**

<u>Solution:</u>

$$\frac{dy}{dx} = \frac{d}{dx}(e^{7x} + sin3x + e^{5x+3}.) = 7e^{7x} + 3cos3x + 5e^{5x+3}$$

****

- **Find $\frac{dy}{dx}$ if $y = log_7 x$.**

<u>Solution:</u> we know that ,if $y = log_a x$ then $\frac{dy}{dx} = \frac{1}{x} log_a e$

$$\frac{dy}{dx} = \frac{1}{x} log_7 e$$

****

<u>Chain Rule (or) Differential coefficient of a function of function</u>

If u=f(x), y=f(u) then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

- **Find $\frac{dy}{dx}$, if $y = x + \frac{1}{x} + \left(x + \frac{1}{x}\right)^3$. (L1)**

<u>Solution:</u>

$$\frac{dy}{dx} = 1 + (-1)x^{-2} + 3\left(x + \frac{1}{x}\right)^2 \frac{d}{dx}\left(x + \frac{1}{x}\right)$$

$$= 1 - \frac{1}{x^2} + 3\left(x + \frac{1}{x}\right)^2\left(1 - \frac{1}{x^2}\right)$$

****

- **Find $\frac{dy}{dx}$,if $y = log_e(2x+3)$.**

Solution:

$$\frac{dy}{dx} = \frac{1}{(2x+3)}\frac{d}{dx}(2x+3)$$

$$= \frac{1}{(2x+3)}(2) = \frac{2}{(2x+3)}$$

****

- **Differentiate $y = cos(x+y)$.**

Solution:

Given, $y = cos(x+y)$

Differentiating both sides w.r.to x

$$\frac{dy}{dx} = -sin(x+y)(1+\frac{dy}{dx})$$

$$= -sin(x+y) - \frac{dy}{dx}(\sin(x+y))$$

$$\frac{dy}{dx}(1+\sin(x+y)) = -\sin(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x+y)}{(1+\sin(x+y))}$$

****

- **Differentiate $y = \log(sin^2 x)$.**

Solution:

Given $y = \log(sin^2 x)$

Let $u = sin^2 x \quad \Rightarrow \quad \frac{du}{dx} = 2\ sinx\ cosx$

$\therefore y = logu \quad \Rightarrow \quad \frac{dy}{du} = \frac{1}{u}$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{1}{u}.2\ sinx\ cosx \quad = \frac{1}{sin^2 x}.\ 2\ sinx\ cosx \quad = 2\frac{cosx}{sinx} = 2cotx$$

****

- **Find $\frac{dy}{dx}$ if $Y = sec(ax+b)$.**

Solution:

Let $y = sec(ax+b)$

$$\frac{dy}{dx} = a\ sec(ax+b)\ tan(ax+b)$$

****

- **Differentiate** $y = \sqrt{2x+3} + e^{\sqrt{tanx}}$.

Solution:

$$y = e^{(tanx)^{1/2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(2x+3)^{-1/2}\,(2) + e^{(tanx)^{1/2}}\frac{d}{dx}(tanx)^{1/2}$$

$$= \frac{1}{\sqrt{2x+3}} + e^{\sqrt{tanx}}\ \frac{1}{2}(tanx)^{-1/2}\frac{d}{dx}(tanx)$$

$$= \frac{1}{\sqrt{2x+3}} + \frac{e^{\sqrt{tanx}}}{2\sqrt{tanx}}\ \sec^2 x$$

****

- **Find** $\frac{dy}{dx}$, *given* $y = \sqrt[3]{x^3+x+1}$.

Solution:

Given $y = \sqrt[3]{x^3+x+1}$

$$\frac{dy}{dx} = \frac{1}{3}[x^3+x+1]^{\frac{1}{3}-1}.\frac{d[x^3+x+1]}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}[x^3+x+1]^{\frac{1}{3}-1}.(3x^2+1)$$

$$= \frac{3x^2+1}{3[x^3+x+1]^{\frac{2}{3}}}$$

****

- **Differentiate** $y = log\left(cos^5(3x^4)\right)$ ***with respect to*** `x'.

Solution:

Given $y = log\left(cos^5(3x^4)\right)$

Differentiate both sides with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\{\log\left(cos^5(3x^4)\right)\}$$

$$= \frac{1}{\left(cos^5(3x^4)\right)} \cdot \frac{d}{dx}\{\left(cos^5(3x^4)\right)\}$$

$$= \frac{1}{\left(cos^5(3x^4)\right)} \frac{d}{dx}\{(\cos(3x^4))^5\}$$

$$= \frac{1}{\left(cos^5(3x^4)\right)} 5(cos(3x^4)^4 \frac{d}{dx}\{\cos(3x^4)\}$$

$$= \frac{5\, cos^4(3x^4)}{\left(cos^5(3x^4)\right)} (-\sin(3x^4))\ \frac{d}{dx}\{(3x^4)\}$$

$$= \frac{5}{\cos 3x^4} (-\sin(3x^4))\{12\, x^3\}$$

$$= (-60\, x^3)\ \frac{\sin(3x^4)}{\cos(3x^4)}$$

$$= -60\, x^3\ \tan 3x^4$$

****

- **Differentiate** $\boldsymbol{y = \left(4x + x^{-5}\right)^{\frac{1}{3}}, with\ respect\ to\ `x'. Hence\ show\ that}$

$\boldsymbol{\frac{dy}{dx} = \frac{4x^6-5}{3x^{\frac{8}{3}}(4X^6+1)^{\frac{2}{3}}}}$.

<u>**Solution:**</u>

$$Given\ \ y = (4x + x^{-5})^{\frac{1}{3}}$$

$$\therefore\ \ \frac{dy}{dx} = \left(\frac{1}{3}\right)(4x + x^{-5})^{\frac{1}{3}-1} \frac{d}{dx}(4x + x^{-5})$$

$$= \left(\frac{1}{3}\right)(4x + x^{-5})^{\frac{-2}{3}}(4 - 5x^{-6})$$

$$= \left(\frac{1}{3}\right)\left(4x + \frac{1}{x^5}\right)^{\frac{-2}{3}}\left(4 - \frac{5}{x^6}\right)$$

$$= \left(\frac{1}{3}\right)\left(\frac{4x^6 + 1}{x^5}\right)^{\frac{-2}{3}}\left(\frac{4x^6 - 5}{x^6}\right)$$

$$= \left(\frac{1}{3}\right)\left(\frac{x^5}{4x^6 + 1}\right)^{\frac{2}{3}}\left(\frac{4x^6 - 5}{x^6}\right)$$

$$= \left(\frac{1}{3}\right) \frac{x^{\frac{10}{3}}}{(4x^6+1)^{\frac{2}{3}}} \left(\frac{4x^6-5}{x^6}\right)$$

$$= \frac{x^{\frac{10}{3}-6}}{3(4x^6+1)^{\frac{2}{3}}}(4x^6-5)$$

$$= \frac{x^{\frac{-8}{3}}(4x^6-5)}{3(4x^6+1)^{\frac{2}{3}}}$$

$$= \frac{4x^6-5}{3x^{\frac{8}{3}}(4X^6+1)^{\frac{2}{3}}}$$

Thus proved.

****

- **Find $\frac{dy}{dx}$ $\boldsymbol{if\ y = log\left[\frac{1+sinx}{1-sinx}\right]}$**

Solution:

Given $y = log\left[\frac{1+sinx}{1-sinx}\right]$

$$\frac{dy}{dx} = \frac{1}{\frac{1+sinx}{1-sinx}}\left\{\frac{d}{dx}\left[\frac{1+sinx}{1-sinx}\right]\right\}$$

$$= \left[\frac{1-sinx}{1+sinx}\right]\left\{\frac{(1-sinx)(cosx)-(1+sinx)(-cosx)}{(1-sinx)^2}\right\}$$

$$= \left\{\frac{cosx - sinxcosx - (-cosx - sinx\ cosx)}{(1+sinx)(1-sinx)}\right\}$$

$$= \frac{2cosx}{1-sin^2x} = \frac{2cosx}{cos^2x} = 2secx$$

****

- **Find $\frac{dy}{dx}$ $\boldsymbol{if\ y = \sqrt{1-sin^2}x}$.**

Solution:

Given $y = \sqrt{1-sin^2}x$

$y=(1-sin^2x)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(1-sin^2x)^{\frac{1}{2}-1}\frac{d}{dx}(1-sin^2x)$$

$$= \frac{1}{2}(1 - sin^2x)^{\frac{-1}{2}}(0 - 2sinx\ cosx)$$
$$= \frac{-sin2x}{2\sqrt{1 - sin^2x}}$$

****

- **Find $\frac{dy}{dx}$ $if\ y = log\left[\frac{(1+\sqrt{x})}{(1-\sqrt{x})}\right]$.**

Solution:

$$y = \ log(1 + \sqrt{x}) - log(1 - \sqrt{x})$$

$$\frac{dy}{dx} = \frac{1}{(1 + \sqrt{x})}\frac{d}{dx}(1 + \sqrt{x}) - \frac{1}{(1 - \sqrt{x})}\frac{d}{dx}(1 - \sqrt{x})$$
$$= \frac{1}{(1 + \sqrt{x})}\ \frac{1}{2\sqrt{x}} - \frac{1}{(1 - \sqrt{x})}\ (\frac{-1}{2\sqrt{x}})$$
$$= \frac{1}{2\sqrt{x}}\left[\frac{1}{(1 + \sqrt{x})} + \frac{1}{(1 - \sqrt{x})}\right]$$
$$= \frac{2}{2\sqrt{x}((1 - (\sqrt{x})^2)} = \frac{1}{\sqrt{x}(1 - x)}$$

****

Differentiation using product rule

Let u & v be differentiable functions of x. Then the product of a function

Y = u(x).v(x) is differentiable.

d(uv)=udv+vdu

- **If $y = x^2 sinx$,find $\frac{dy}{dx}$.**

Solution: By product rule,

$$\frac{dy}{dx} = x^2\frac{d}{dx}(sinx) + sinx\frac{d}{dx}(x^2)$$
$$\frac{dy}{dx} = x^2cosx + 2xsinx$$
$$= x(xcosx + 2sinx).$$

****

- **Find $\frac{dy}{dx}$ if $y=e^x tanx$.**

Solution: Let $y = e^x tanx$

By product rule, $\frac{dy}{dx} = e^x \frac{d}{dx}(tanx) + tanx \frac{d}{dx}(e^x)$

$$\frac{dy}{dx} = e^x \sec^2 x + e^x tanx$$

$$= e^x(\sec^2 x + tanx)$$

****

- **If y=3x^4 e^x ,find $\frac{dy}{dx}$.**

<u>**Solution:**</u>

By product rule, $\frac{dy}{dx} = 3\left(e^x \frac{d}{dx}(x^4) + x^4 \frac{d}{dx}(e^x)\right)$

$$\frac{dy}{dx} = 3(x^4 e^x + 4x^3 e^x)$$

$$=3x^3 e^x(x+4)$$

****

- **If $y = cosx\, e^x$, find $\frac{dy}{dx}$.**

<u>**Solution:**</u>

By product rule, $\frac{dy}{dx} = e^x \frac{d}{dx}(cosx) + cosx \frac{d}{dx}(e^x$

$$\frac{dy}{dx} = e^x(-sinx) + e^x cosx \quad = e^x(cosx - sinx)$$

****

- **If y= $xlog_e x$, find $\frac{dy}{dx}$.**

<u>**Solution:**</u>

By product rule, $\frac{dy}{dx} = x \frac{d}{dx}(log_e x) + log_e x \frac{d}{dx}(x)$

$$\frac{dy}{dx} = x\frac{1}{x} + log_e x$$

$$=1+ \ log_e x$$

****

- **Differentiate Y=$(x^2 - 2)\ (3x + 1)$. (L4)**

<u>**Solution:**</u>

Let y= $(x^2 - 2)\ (3x+1)$.

$$\frac{dy}{dx} = (x^2 - 2)3 + (3x+1)(2x)$$

$= 3\,(x^2 - 2) + 2x\,(3x + 1)$
$= 3x^2 - 6 + 6x^2 + 2x$
$= 9x^2 + 2x - 6$

****

- **If $\boldsymbol{y = cosecx\ cotx}$, find $\boldsymbol{\frac{dy}{dx}}$.**

<u>**Solution:**</u>

By product rule, $\frac{dy}{dx} = cosecx\frac{d}{dx}(cotx) + cotx\frac{d}{dx}(cosecx)$

$$\frac{dy}{dx} = cosecx(-\operatorname{cosec}^2 x) + cotx\ (-cosecx\ cotx)$$

$$= -\ cosecx\ (\ cosec^2 x +\ \ cot^2 x)$$

****

- **If y= $\boldsymbol{(x^2 + 7x + 2)\ (e^x - logx)}$, find$\boldsymbol{\frac{dy}{dx}}$.**

<u>**Solution:**</u>

By product rule,

$$\frac{dy}{dx} = (x^2 + 7x + 2)\ \frac{d}{dx}(e^x - logx) + (e^x - logx)\frac{d}{dx}(x^2 + 7x + 2$$

$$= (x^2 + 7x + 2)(\ e^x - \tfrac{1}{x}) + (e^x - logx)(2x + 7),$$

****

- **If $\boldsymbol{y = (6sinx\ log_{10}x\ )}$,find $\boldsymbol{\frac{dy}{dx}}$.**

<u>**Solution:**</u>

By product rule, $\frac{dy}{dx} = 6\left\{sinx\frac{d}{dx}(log_{10}x\ ) + log_{10}x\ \frac{d}{dx}(sinx)\right\}$

$$= 6\left\{sinx\left(\frac{1}{x}\right)log_{10}e + log_{10}x(cosx)\right\}$$

****

- **If $\boldsymbol{y=(e^x\ logxcotx)}$,find $\boldsymbol{\frac{dy}{dx}}$.**

<u>**Solution:**</u>

By product rule,

$$\frac{dy}{dx} = e^x logx\frac{d}{dx}(cotx) + e^x\ cotx\frac{d}{dx}(logx) + logxcotx\frac{d}{dx}(e^x\ )$$

$$= e^x\ logx(-\operatorname{cosec}^2 x) + e^x\ cotx\left(\frac{1}{x}\right)logx\ cotx\ e^x.$$

****

- **Differentiates$\boldsymbol{sin^2x\ cos3x}$.**

<u>**Solution:**</u>

Let $y = sin^2x\ cos3x$

$$\frac{dy}{dx} = sin^2x\ \frac{d}{dx}(cos3x) + cos3x\ \frac{d}{dx}(sin^2x)$$

$$= sin^2x\ (-sin3x).3 + cos3x[2sinx.\frac{d}{dx}(sinx)]$$

$$= -3sin^2x\ sin3x + cos3x\ [2sinx\ cosx]$$

$$= sinx[-3sinx\ sin3x + 2cosx\ cos3x]$$

****

- **Differentiate $\boldsymbol{e^{4x}sin4x}$.**

<u>**Solution:**</u>

Let $y = e^{4x}sin4x$

$$\frac{dy}{dx} = e^{4x}\ \frac{d}{dx}(sin4x) + sin4x\frac{d}{dx}(e^{4x})$$

$$= e^{4x}[cos4x]\frac{d}{dx}(4x) + sin4x[e^{4x}.\frac{d}{dx}(4x)]$$

$$= e^{4x}[cos4x](4) + sin4x[e^{4x}.4]$$

$$= 4e^{4x}[cos4x + sin4x]$$

****

<u>**Quotient Rule for Differentiation**</u>

Let u & v be differentiable functions of x, then $\frac{u}{v}$ is also differentiable

$d(\frac{u}{v}) = \frac{vu' - uv'}{v^2}$

- **Find y' , if y=$\boldsymbol{\frac{x^2}{1+x^2}}$.**

<u>**Solution:**</u>

By quotient rule, $\frac{dy}{dx} = \frac{(1+x^2)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(x^2+1)}{(1+x^2)^2}$

$$\frac{dy}{dx} = y' = \frac{(1+x^2)(2x) - x^2(2x)}{(1+x^2)^2} = \frac{1}{(1+x^2)^2}$$

****

- **Find y' , if y=** $\frac{x^2-1}{1+x^2}$.

Solution:

By quotient rule,

$$\frac{dy}{dx} = \frac{(1+x^2)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x^2+1)}{(1+x^2)^2}$$

$$\frac{dy}{dx} = y' = \frac{(1+x^2)(2x) - (x^2-1)(2x)}{(1+x^2)^2} = \frac{4x}{(1+x^2)^2}$$

****

- **Find y' , if y=** $\frac{2x-3}{4x+5}$.

Solution:

By quotient rule ,

$$\frac{dy}{dx} = \frac{(4x+5)\frac{d}{dx}(2x-3) - (2x-3)\frac{d}{dx}(4x+5)}{(4x+5)^2}$$

$$\frac{dy}{dx} = y' = \frac{(4x+5)(2) - (2x-3)4}{(4x+5)^2}$$

$$= \frac{8x+10-8x+12}{(4x+5)^2}$$

$$= \frac{22}{(4x+5)^2}$$

- **Find y' , if y=** $\frac{logx}{sinx}$.

Solution:

By quotient rule

$$\frac{dy}{dx} = \frac{(sinx)\frac{d}{dx}(logx) - (logx)\frac{d}{dx}(sinx)}{(sinx)^2}$$

$$\frac{dy}{dx} = y' = \frac{sinx\left(\frac{1}{x}\right) - logx(cosx)}{(sinx)^2}$$

$$= \frac{sinx - x\ cosx\ logx}{x\ \sin^2 x}$$

****

- **Find y' , if $y=\frac{logx^2}{e^x}$.**

<u>**Solution:**</u>

$$y = \frac{2logx}{e^x}$$

By quotient rule,

$$\frac{dy}{dx} = \frac{(e^x)\frac{d}{dx}(2logx) - (2logx)\frac{d}{dx}(e^x)}{(e^x)^2}$$

$$\frac{dy}{dx} = y' = \frac{e^x\left(2\frac{1}{x}\right) - 2logx(e^x)}{(e^x)^2} = \frac{2e^x - x(2logx(e^x))}{x(e^x)^2}$$

$$= \frac{2e^x(1 - \text{xlogx})}{xe^{2x}}$$

****

- **Find y' , if $y=\frac{x^2+e^x}{(cosx+logx)}$.**

<u>**Solution:**</u>

By quotient rule,

$$\frac{dy}{dx} = \frac{(cosx + logx)\frac{d}{dx}(x^2 + e^x) - (x^2 + e^x)\frac{d}{dx}(cosx + logx)}{(cosx + logx)^2}$$

$$\frac{dy}{dx} = y' = \frac{(cosx + logx)(2x + e^x) + (x^2 + e^x)\left(-sinx + \frac{1}{x}\right)}{(cosx + logx)^2}$$

****

- **Find y' , if $y=\frac{sinx+cosx}{sinx-cosx}$.**

<u>**Solution:**</u>

By quotient rule,

$$\frac{dy}{dx} = \frac{(sinx - cosx)\frac{d}{dx}(sinx + cosx) - (sinx + cosx)\frac{d}{dx}(sinx - cosx)}{(sinx - cosx)^2}$$

$$\frac{dy}{dx} = y' = \frac{(sinx - cosx)(cosx - sinx) - (sinx + cosx)(cosx + sinx)}{(sinx - cosx)^2}$$

$$= \frac{-(sinx - cosx)^2 - (sinx + cosx)^2}{(sinx - cosx)^2}$$

****

- **Differentiate $y = \frac{(x^7 - 4^7)}{(x-4)}$.**

Solution:

By quotient rule,

$$\frac{dy}{dx} = \frac{(x-4)\frac{d}{dx}(x^7 - 4^7) - (x^7 - 4^7)\frac{d}{dx}(x-4)}{(x-4)^2}$$

$$\frac{dy}{dx} = y' = \frac{(x-4)(7x^6) - (x^7 - 4^7)(1)}{(x-4)^2}$$

$$= \frac{7x^7 - 28x^6 - x^7 + 4^7)}{(x-4)^2} = \frac{(6x^7 - 28x^6 + 4^7)}{(x-4)^2}$$

****

- **Differentiate $Y = \frac{(x+1)}{(x^2+1)}$.**

Solution:

$$y = \frac{(x+1)}{(x^2+1)}$$

$$\frac{dy}{dx} = \frac{(x^2+1) - (x+1)2x}{(x^2+1)^2}$$

$$= \frac{x^2 + 1 - 2x^2 - 2x}{(x^2+1)^2} = \frac{-x^2 - 2x + 1}{(x^2+1)^2}$$

****

- **Differentiate $\frac{x^2 - x + 1}{x^2 + x + 1}$**

Solution:

Let $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$$\frac{dy}{dx}=\frac{(x^2+x+1)(2x-1)-(x^2-x+1)(2x+1)}{(x^2+x+1)^2}$$

$$=\frac{(2x^3-x^2+2x^2-x+2x-1)-(2x^3+x^2-2x^2-x+2x+1)}{(x^2+x+1)^2}$$

$$=\frac{2x^3+x^2+x-1-2x^3+x^2-x-1}{(x^2+x+1)^2}$$

$$=\frac{2x^2-2}{(x^2+x+1)^2} \quad =\frac{2(x^2-1)}{(x^2+x+1)^2}$$

****

- **Differentiate $\frac{secx}{logx}$.**

Solution:

Let $y=\frac{secx}{logx}$

$$\frac{dy}{dx}=\frac{logx(secx\,tanx)-secx.\frac{1}{x}}{(logx)^2}$$

$$=\frac{x\,logx\,secx\,tanx-secx}{x(logx)^2}$$

$$=\frac{secx[xtanx\,logx-1]}{x(logx)^2}$$

****

- **Find the derivative of $\frac{e^x+e^{-x}}{e^x-e^{-x}}$.**

Solution:

Let $y=\frac{e^x+e^{-x}}{e^x-e^{-x}}$

$$\frac{dy}{dx}=\frac{(e^x-e^{-x})(e^x-e^{-x})-(e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}$$

$$=\frac{(e^x-e^{-x})^2-(e^x+e^{-x})^2}{(e^x-e^{-x})^2}$$

$$=\frac{(e^{2x}+e^{-2x}-2)-(e^{2x}+e^{-2x}+2)}{(e^x-e^{-x})^2}$$

$$=\frac{(e^{2x}+e^{-2x}-2-e^{2x}-e^{-2x}-2)}{(e^x-e^{-x})^2}=\frac{-4}{(e^x-e^{-x})^2}$$

****

- **Differentiate** $y = \frac{te^{2t}}{2\,cost}$ ***with respect to*** $`t'$.

<u>Solution:</u>

$$\text{Given} \quad y = \frac{te^{2t}}{2\ cost}$$

$$\text{Let } u = te^{2t}, \qquad v = 2\ cost$$

$$\frac{du}{dt} = (t)(2e^{2t}) + (e^{2t})(1) = 2te^{2t} + e^{2t}, \qquad \frac{dv}{dt} = -2sint$$

$$\therefore \ \frac{dy}{dt} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(2cost)[2te^{2t} + e^{2t}] - (te^{2t})(-2sint)}{(2\ cost)^2}$$

$$= \frac{4t\ e^{2t}cost + 2e^{2t}cost + 2t\ e^{2t}sint}{4cos^2t}$$

$$= \frac{2e^{2t}[2t\ cost + cost + t\ sint]}{4cos^2t}$$

$$(i.e) \quad \frac{dy}{dt} = \frac{e^{2t}}{2cos^2t}(2t\ cost + cost + t\ sint)$$

****

<u>**Differentiation of Parametric functions**</u>

If x and y are expressed in terms of a third variable t , then the third variable is called the parameter, equation containing a parameter is known as parametric equation.

(ie) If x = f(t), y = g(t) then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

- **If** $\boldsymbol{x = \sqrt{t}, y = t + \frac{1}{t}}$ **,then find** $\boldsymbol{\frac{dy}{dx}}$**.**

  <u>**Solution:**</u>

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$x = \sqrt{t} \quad \Longrightarrow \quad \frac{dx}{dt} = \frac{1}{2\sqrt{t}}$$

$$y = t + \frac{1}{t} \quad \Rightarrow \quad \frac{dy}{dt} = 1 - \frac{1}{t^2}$$

$$\frac{dy}{dx} = \frac{(1 - \frac{1}{t^2})}{\frac{1}{2\sqrt{t}}} = \frac{2(t^2 - 1)\sqrt{t}}{t^2} = 2\frac{(t^2 - 1)}{t^{\frac{3}{2}}}$$

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- If $\boldsymbol{x = a(1 + cos\theta)}$; $\boldsymbol{y = a(\theta + sin\theta)}$ **then, find** $\boldsymbol{\frac{dy}{dx}}$**.**

  Solution:

$$\frac{dx}{d\theta} = a(0 - Sin\theta) = -a \sin\theta$$

$$\frac{dy}{d\theta} = a(1 + cos\theta)$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\mathrm{a}(1 + \cos\theta)}{-\mathrm{a}\sin\theta}$$

$$= \frac{-\cos^2\frac{\theta}{2}}{sin\frac{\theta}{2}cos\frac{\theta}{2}}$$

$$= -cot\frac{\theta}{2}$$

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- **Find** $\boldsymbol{\frac{dy}{dx}}$ $\boldsymbol{, if\ x = a(t - sint), y = a(1 - cost)}$**.**

Solution:

Given $x = a(t - sint)$, $\quad y = a(1 - cost)$

$$\frac{dx}{dt} = a(1 - cost) \quad , \quad \frac{dy}{dt} = a(0 + sint)$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a(sint)}{a(1 - cost)} = \frac{sint}{1 - cost}$$

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- **Find $\frac{dy}{dx}, if\ \ x = ct\ , y = \frac{c}{t}$.**

<u>Solution:</u>

$$\text{Given}\quad x = ct\ ,\qquad y = \frac{c}{t}$$

$$\frac{dx}{dt} = c\ ,\qquad \frac{dy}{dt} = \frac{c(-1)}{t^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy/_{dt}}{dx/_{dt}} = -\frac{c}{t^2} \times \frac{1}{c} = -\frac{1}{t^2}$$

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- **Find $\frac{dy}{dx}, if\ \ x = acost, y = bsint$.**

<u>Solution:</u>

$$\text{Given}\ \ x = acost,\qquad y = bsint$$

$$\frac{dx}{dt} = -a\ sint\ ,\qquad \frac{dy}{dt} = b\ cost$$

$$\therefore \frac{dy}{dx} = \frac{dy/_{dt}}{dx/_{dt}} = \frac{b\ cost}{-a\ sint} = -\frac{b}{a}cot\ t$$

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- **Find $\frac{dy}{dx}, if\ \ x = acos^2t, y = bsin^2t$.**

<u>Solution:</u>

$$\text{Given}\ \ x = acos^2t,\qquad y = bsin^2t$$

$$\frac{dx}{dt} = a\ 2cost(-\ sint)\ ,\qquad \frac{dy}{dt} = 2bsint\ cost$$

$$\therefore \frac{dy}{dx} = \frac{dy/_{dt}}{dx/_{dt}} = \frac{2\ b\ sint\ cost}{-2\ a\ cost\ sint} = -\frac{b}{a}$$